Understanding subluminal and superluminal propagation through superposition of frequency components

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Propagation of a light pulse through a dielectric slab is discussed theoretically in this paper. It is exhibited via a multiple-scattering approach that the slab can modify the phase of the pulse's frequency components, so that, when the frequency components are superposed, they cause the peak in the output pulse to appear to travel either faster or slower than in vacuum, depending on whether the slab is absorbing or amplifying. The expressions of the corresponding advancement and delay of the peak are derived and argued to be limited in magnitude by the pulse's duration.

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Although the problem of propagation of a light pulse inside dispersive media has been studied for more than a century, it still attracts considerable interest. This is largely because of the discovery that the velocity of the pulse, assumed to be the group velocity v_g [1], can be either larger [2] (superluminal propagation) or smaller [3] (subluminal propagation) than the speed of light in vacuum c in certain situations while the pulse itself suffers negligible distortion [4]. In particular, it has been predicted that the peak of a pulse can emerge from a dielectric slab at an instant even earlier than the instant at which the peak of the pulse enters the slab [5]. Recently, possible applications of superluminal propagation of light in the information theory are studied [6,7].

Of course, even $v_g > c$ does not imply the violation of causality [8]. One way (see Refs. [4,9,10], for other possible explanations) to understand this is to note that any pulse is composed of a spread of frequency components and at the peak of the pulse these frequency components are all in phase; after the pulse's transmission through a medium the relative phases among the frequency components are modified, so that a coherent superposition of these components gives rise to a shifted peak in the output pulse, causing the pulse to appear to travel at a speed different from c [4]. Thus, under this viewpoint, considerable amount of the information regarding the propagation of a light pulse through a dispersive medium, a dielectric slab for example, can be obtained merely through an analysis of the phase of the frequency components. In the present paper, it is demonstrated that the phase analysis can not only explain the phenomena of superluminal and subluminal propagation of a light pulse through a dielectric slab but also enable one to derive the associated peak advancement (delay) when the slab is absorbing (amplifying).

It is worth while to note that, although superluminal and subluminal propagation of a light pulse have been known for many years, it is still unclear what physical processes can lead to a limitation on the total advancement (delay) that the pulse can experience [11]. In this paper, it is shown that such a limitation actually comes from the pulse's duration.

Light propagation is formulated in the present work using the theory of multiple scattering. This approach can not only treat the phase of any frequency component accurately (see Ref. [12] for an illustration) but also conveniently describe the transient nature of light propagation [13].

Consider a light pulse with a duration of τ (including the pulse's leading and trailing wings) incident at a time $t=0$ from the $z<0$ region on a one-dimensional slab situated between $z=0$ and $z=d$. The slab is assumed to be composed of uniformly distributed identical atoms whose polarizability $\alpha=\gamma_0/(\omega-\omega_0+i\gamma)$ is in near resonance with the pulse, where ω_0 is the oscillation frequency of the atoms, and γ is a positive constant that represents the linewidth of the atomic transition. The slab is absorbing when γ_0 in the expression of α is negative and amplifying when γ_0 is positive [14]. The peak of the incident pulse is chosen to be at the center of the pulse. This choice is achievable by requiring, for example, the front edge of an arbitrary frequency component ω to have a phase $e^{i\omega\tau/2}$ (evaluated at $z=0$), so that the phase of this component at a different time t becomes $e^{i\omega\tau/2-i\omega t}$ (still evaluated at $z=0$), and, when $t=\tau/2$, all the frequency components have a null phase at $z=0$.

Time evolution of the light inside the slab is governed by the following wave equation of the electric field E:

$$\frac{\partial^2}{\partial z^2}E(z,t)-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}E(z,t)=\frac{4\pi n\alpha}{c^2}\frac{\partial^2}{\partial t^2}E(z,t), \tag{1}$$

where n is the density of the atoms, and the slab is assumed to be nonmagnetic. The phase modification of the frequency component ω after the pulse passes through the slab can be accounted for with the help of a multiple-scattering reformulation of the preceding differential equation, which shows that the electric field at $z=R$ ($R>d$) can be expressed as a series:

$$E(R,t)=e^{ikR-i\omega t+i\omega\tau/2}+\beta\int_0^d e^{ik|R-z_1|+ikz_1+i\omega\tau/2-i\omega t}dz_1 + \beta^2\int_0^d dz_1\int_0^d dz_2 e^{ik|R-z_1|+ik|z_1-z_2|+ikz_2+i\omega\tau/2-i\omega t}+\cdots, \tag{2}$$

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where the first term on the right-hand side (RHS) represents that the frequency component goes directly to an observation point R, and the following terms indicate respectively that the frequency component experiences scattering from the slab once, twice,..., before arriving at the observation point. Also in the preceding equation, the symbol β is used to denote $i2\pi n\alpha\omega/c$, and $k=\omega/c$ [15]. If backscattering is included in Eq. (2), the frequency component will spend more time in the slab and suffer additional distortion that comes from a source different from dispersion: Every part of the frequency component, except the front edge, will be added with its preceding parts scattered backward. Since its lowest order in the present one-dimensional case is proportional to β^2 and becomes insignificant in magnitude when n is small, the backscattering will be ignored in the discussion to follow.

To consider only forward scattering amounts to requiring scattering events to be arranged successively along the increasing direction of z. For example, in the third term on the RHS of Eq. (2), one should restrict $0 \leqslant z_2 \leqslant z_1 \leqslant d$, so that the second scattering event takes place at a point more distant from the vacuum-slab interface $z=0$ than the first scattering event does. If the dwell time of light at each scattering step is ignored too, the series in Eq. (2) shows that the light pulse propagates in fact at the speed c inside the medium, not just its front edge [8]. A subsequent calculation then shows that the series in Eq. (2) can be summed up into closed form

$$E(R,t) = e^{\beta d + i\omega\tau/2 + ikR - i\omega t}. \tag{3}$$

Note that the expression in Eq. (3) is not the usual expression of a plane wave transmitted through the slab. The latter expression is often obtained by first solving Maxwell equations in the frequency domain subject to boundary conditions and then performing the inverse Fourier transform to add the time variable; see Ref. [16] for instance. But it can be shown that the frequency domain solution is actually equivalent to a multiple-scattering process, which corresponds to including every order of scattering, both forward and backward [17], and, therefore, to represent a situation where the transmitted field is observed for an infinitely long time such that all orders of scattering are registered. To formulate light propagation through a slab, which is a transient process in nature [13], clearly only forward-scattering terms are needed in Eq. (2).

Since the light still propagates at the speed c even inside the slab, the relation established in Eq. (3) has to be interpreted as follows. At the observation point $z=R$, no signal is recorded before the time $t=R/c$; after that time, although the light pulse appears, its amplitude and phase are both altered by the slab through βd. By simply ignoring ikR in Eq. (3), one can formally treat t in the equation as starting from 0 rather than from R/c. The expression of α is substituted in βd to illustrate the latter quantity as a function of the frequency ω:

$$\beta d \simeq \frac{2\pi n\omega_0 d\gamma_0}{c}\left(\frac{1}{\gamma} + i\frac{\omega-\omega_0}{\gamma^2}\right), \tag{4}$$

where an assumption that the spectral width of the pulse around ω_0 is much smaller than that of the atomic transition, $|\omega_0-\omega| \leqslant \gamma$, has been made; such a condition is often assumed in theoretical discussions [5,14] and achieved in experiments [18] for the reason that only when the light pulse is largely in resonance with the slab will the anomalous dispersion become significant. The expression in Eq. (3) subsequently reduces to

$$E(t) = e^{2\pi n\omega_0 d\gamma_0/c\gamma - i2\pi n\omega_0^2 d\gamma_0/c\gamma^2 + i\omega(\tau/2 - t + 2\pi n\omega_0 d\gamma_0/c\gamma^2)}. \tag{5}$$

When $\gamma_0 < 0$, the slab is absorbing, and the magnitude of E is noted to be reduced by a factor of $\exp(2\pi n\omega_0 d\gamma_0/c\gamma)$. An examination of the expression of α reveals that the reduction is entirely caused by the imaginary part of the polarizability. The real part of the same polarizability, on the other hand, plays a different role: It modifies the pulse's phase in such a way that the time (relative to the pulse's front edge) that takes all the frequency components to reach a common phase changes from the initial $\tau/2$ (corresponding to the common phase 0) to the present $\tau/2 + 2\pi n\omega_0 d\gamma_0/(c\gamma^2)$ [corresponding to another common phase $-i2\pi n\omega_0^2 d\gamma_0/(c\gamma^2)$]. Therefore, in the observation region $z > d$, the peak of the pulse advances by $\Delta T \equiv 2\pi n\omega_0 d|\gamma_0|/(c\gamma^2)$, and the pulse thus appears to exercise superluminal propagation. As the present discussion shows, the real physics behind this phenomenon is phase modification of the frequency components. Similarly, when $\gamma_0 > 0$, the slab becomes amplifying, and the peak of the pulse will be delayed by ΔT. Such an effect is referred to as subluminal propagation.

In a higher-dimensional space, wave superposition is well known to be responsible for the Goos-Hänchen shift that takes place, for example, after a light beam transmits through a dielectric slab; see Ref. [19]. In the present one-dimensional discussion, clearly, no such shift exists, because the superposition of the frequency components can only cause the peak of the pulse to move in the z direction.

With the help of ΔT, it is then straightforward to derive that, when $\gamma_0 < 0$, the peak formally travels inside the slab at a speed $c/(1 - c\Delta T/d)$, and, when $\gamma_0 > 0$, at a different speed $c/(1 + c\Delta T/d)$. These two speeds are exactly equal to the group velocity of the pulse evaluated in these two situations inside the slab, even though as pointed out earlier the pulse itself propagates at the speed of c. Thus, the pulse and its peak actually travel inside the slab at different speeds.

For a slab, whose d is small, the pulse is largely able to keep its initial shape after transmission through the slab. For a slab with a large width, on the other hand, the pulse not only suffers a more significant change in magnitude but also is expected to experience a large deformation due to the increment in ΔT [see the expression in Eq. (5)], for the reason that, if $\Delta T > \tau/2$ (either advancement or delay), it will be impossible for the frequency components to achieve a common phase throughout the duration of the pulse, and the peak will simply disappear in the output pulse as a result of the destructive superposition of the frequency components. (Note that in the present discussion the peak is originally assumed to be at the center of the pulse.) This observation effectively sets an upper limit on ΔT, i.e., $\Delta T \leqslant \tau/2$. A different prediction is reported in Ref. [11], where the authors conclude, based on a group-velocity analysis, that no limitation exists on the maximum achievable time delay of a light pulse. Such a conclusion has to be treated with caution, because it is derived under the authors' assumption of a Lorentzian dispersion relation, which is in fact not compatible with the model adopted in the reference.

In conclusion, the phenomena of superluminal and subluminal propagation of a light pulse through a dielectric slab are explained, through explicit calculations, as resulting from the phase change of the frequency components that form the light pulse. Both the advancement and delay of the pulse's peak are calculated and found to be limited by the pulse's duration.

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